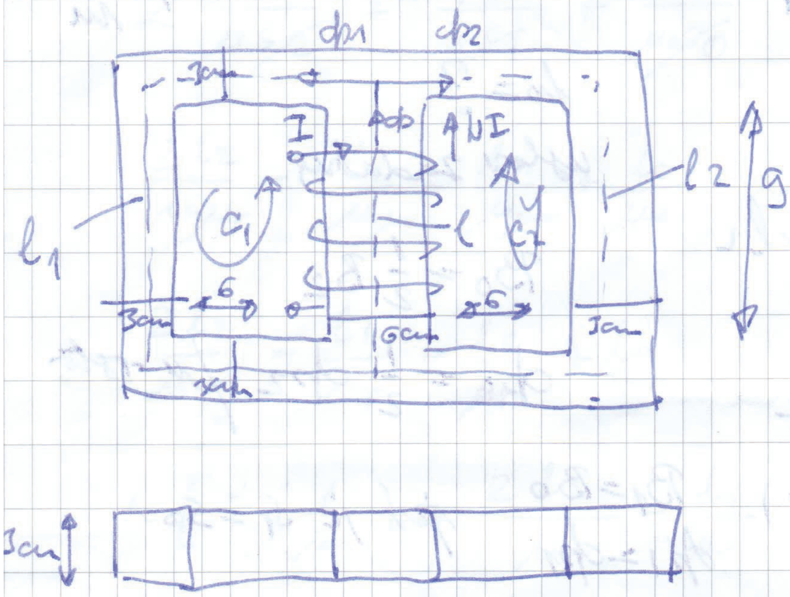
5.3. Linearno magnetno kolo ima oblik i dimenzije kao na slici. Na srednjem dijelu kola je namotano $N = 100$ navojaka sa strujom 2A. Relativna magnetna permeabilnost je $\mu_r = 500$.

a) Odrediti vrijednost Φ, B i H u svim granama kola

b) Ako se u jednoj od bočnih grana napravi vazdušni procjep, kolika treba da bude njegova dužina pa da indukcija u procjepu bude dva puta manja nego u drugoj bočnoj grani.



$l_1 = l_2 = 33cm$

$l = 12cm$

$S_1 = S_2 = 9cm^2$

$S = 18cm^2$

$\mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$

$H = \frac{B}{\mu}$

$R_{m1} = \frac{l_1}{\mu S_1}$ $\quad S_1 = S_2$

$R_{m2} = \frac{l_2}{\mu S_2}$ $\quad \boxed{R_{m1} = R_{m2}}$ (*)

$R_m = \frac{l}{\mu S}$

$\Sigma \Phi = 0 \qquad \Phi = \Phi_1 + \Phi_2 \quad (1)$

$\Sigma NI = \Sigma R_m \Phi$

$R_m \Phi + R_{m1} \Phi_1 = NI \quad (2)$

$R_m \Phi + R_{m2} \Phi_2 = NI \quad (3)$

(2) − (3) $\quad R_{m1} \Phi_1 = R_{m2} \Phi_2 \quad (4)$

iz (*) i (4): $\boxed{\Phi_1 = \Phi_2} \quad (5)$

(5) u (1) $\quad \Phi = 2\Phi_1 \quad (6) \quad \Phi_1 = \frac{\Phi}{2}$

(6) u (2) $\quad R_m \Phi + R_{m1} \frac{\Phi}{2} = NI \Rightarrow \Phi = \frac{NI}{R_m + \frac{R_{m1}}{2}}$

$\Phi_1 = \Phi_2 = \frac{\Phi}{2} = 2{,}5 \cdot 10^{-4} Wb$

$\Phi = 5 \cdot 10^{-4} Wb$

nastavak

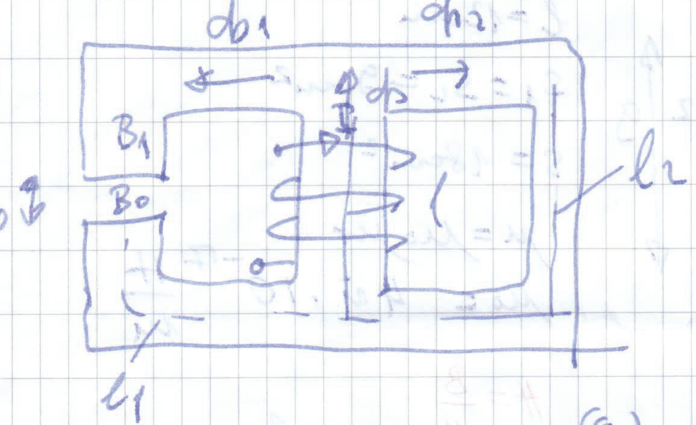
$\phi = B \cdot S$

$B = \frac{\phi}{S} = 0{,}278\ T$

$B_1 = B_2 = \frac{\phi_1}{S_1} = 0{,}278\ T$

$H = \frac{B_1}{\mu} \quad H_1 = \frac{B_1}{\mu} \quad H_2 = \frac{B_2}{\mu} \Rightarrow H_1 = H_2 = H = 442{,}5\ \frac{A}{m}$

b)



$l_0 = ?$

uslov zadatka

$B_0 = \frac{1}{2} B_2$

$\phi_0 = \frac{1}{2} \phi_2$ / μ-isto

(a) $B_1 = B_0$ jer je $S_1 = S_0$

$\phi_1 = \phi_1$

$B_1 \cdot S_1 = B_0 S_0$

$B_1 = B_0$

$\phi = \phi_1 + \phi_2 \quad (1)$

$NI = \phi R_m + \phi_1 R_{m1}' + \phi_1 R_{m0} \quad (2)$

$NI = R_m \phi + R_{m2} \phi_2 \quad (3)$

$\phi_2 \neq \phi_1$

(2) i (3) $\quad \phi_1 (R_{m1}' + R_{m0}) = \phi_2 R_{m2} \quad (*)$

iz (a) $\quad B_2 = 2 B_0 \Rightarrow \quad \frac{\phi_2}{S_2} = 2 \frac{\phi_0}{S_0}$

$S_2 = S_0 \Rightarrow$

(c) $\quad \phi_2 = 2\phi_0 = 2\phi_1$

$R_{m1}' = \frac{l_1 - l_0}{\mu S_1}$

$R_0 = \frac{l_0}{\mu S_1}$

$R_{m2} = \frac{l_2}{\mu S_2}$

$$\Phi_1 (R_{m1}' + R_{m0}) = \Phi_2 R_{m2}$$

$$\Phi_2 = 2\Phi_0 = 2\Phi_1$$

$$\Phi_1 (R_{m1}' + R_{m0}) = 2\Phi_1 R_{m2}$$

$$2 R_{m2} = R_{m1}' + R_{m0}$$

$$2 \frac{l_2}{\mu S_2} = \frac{l_1 - l_0}{\mu S_1} + \frac{l_0}{\mu_0 S_0} \qquad S_1 = S_2 = S_0$$

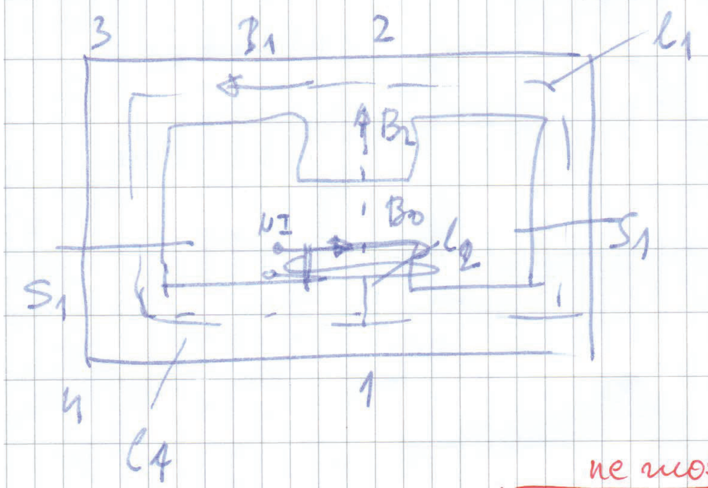
$$\frac{2 l_2}{\mu_0 \mu_r} = \frac{l_1}{\mu_0 \mu_r} - \frac{l_0}{\mu_0 \mu_r} + \frac{l_0}{\mu_0} \qquad / \mu_0$$

$$\frac{2 l_2}{\mu_r} = \frac{l_1}{\mu_r} - \frac{l_0}{\mu_r} + l_0$$

$$l_0 \left(1 - \frac{1}{\mu_r}\right) = \frac{2 l_2}{\mu_r} - \frac{l_1}{\mu_r} = \frac{2 l_2 - l_1}{\mu_r}$$

$$l_0 = \frac{2 l_2 - l_1}{\mu_r - 1} = 0{,}66 \text{ mm}$$

5.7. Dato je simetrično magnetno kolo kao na slici. Poznate su dimenzije $l_1=30\,cm$; $l_2=10\,cm$, $S_1=10\,cm^2$ i $S_2=25\,cm^2$, $d=1\,mm$. Ako je vrijednost indukcije u ~~prostoru~~ procjepu $B_0=0,5\,T$, odrediti vrijednost indukcije u ostalim dijelovima kola i magnetnu pobudnu silu namotaja NI . $\mu_r = 400$



$l_1 = 30\,cm$

$l_2 = 10\,cm$

$S_1 = 10\,cm^2$

$S_2 = 25\,cm^2$

$d = 1\,mm$

$B_0 = 0,5\,T$

$NI = ?$ $B_1 = ?$ $B_2 = ?$

$$\oint_L \vec{H}\,d\vec{l} = \Sigma NI$$

$$\oint_L \vec{B}\,d\vec{l} = \mu NI$$

ne može! radi μ!

μ je različito od. μ_0 i μ_r

$H = \frac{B}{\mu}$

granični uslov (2) $B_2 = B_0$

$$H_2 l_2 + H_0 l_0 + H_1 l_1 = NI$$

$$\frac{B_2}{\mu} l_2 + \frac{B_0}{\mu_0} d + \frac{B_1}{\mu_1} l_1 = NI \quad (1)$$

$\oint_S B\,dS = 0$ zakon o održanju mag. fluksa

$$B_2 S_2 = B_1 S_1 + B_1 S_1 \Rightarrow B_2 S_2 = 2 B_1 S_1$$

$$B_1 = \frac{B_2 S_2}{2 S_1} \quad (3)$$

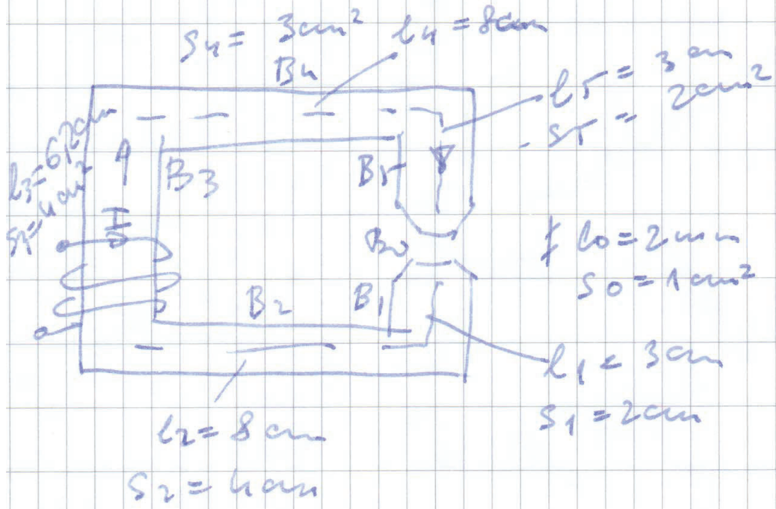
(2) i (3) u (1) ~~$\frac{B_2}{\mu} l_2$~~ $\frac{B_0 d}{\mu_0} + \frac{B_0}{\mu} l_2 + \frac{S_2 B_0}{2 S_1} = NI$

$NI = 634$ Amav.

$B_2 = B_0 = 0,5\,T$

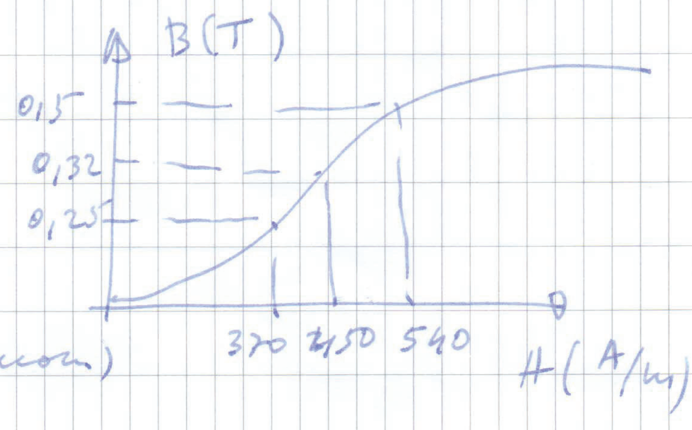
$B_1 = \frac{B_0 S_2}{2 S_1} = 1,25 \cdot 0,5\,T = 0,625\,T$

5.12. Magnetno kolo prema slici napravljeno je od feromagnetnog materijala. Izračunati jačinu struje koja treba da protiče kroz namotaj da bi indukcija u vazdušnom procjepu bila $B_0 = 1T$



$I = ?$

$B_0 = 1T$



A. zakon (ne može K-H. zakon)

$$H_1 l_1 + H_2 l_2 + H_3 l_3 + H_4 l_4 + H_5 l_5 + H_0 l_0 = NI \quad (*)$$

$$H_0 = \frac{B_0}{\mu_0} = \frac{1}{4\pi \cdot 10^{-7}} \frac{A}{m}$$

B ide normalno na graničnu površinu | sa slike

$\Phi_0 = \Phi_1 \quad B_0 S_0 = B_1 S_1 \Rightarrow B_1 = \frac{B_0 S_0}{S_1} = \frac{B_0}{2} \rightarrow H_1 = 540 \frac{A}{m}$

$\Phi_2 = \Phi_1 \Rightarrow B_2 S_2 = B_1 S_1 \Rightarrow B_2 = \frac{B_1 S_1}{S_2} = \frac{B_0}{4} \rightarrow H_2 = 370 \frac{A}{m}$

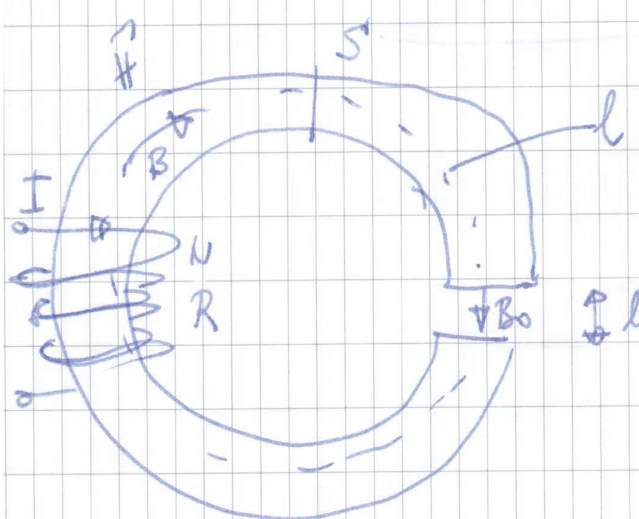
$\Phi_3 = \Phi_2 \Rightarrow B_3 S_3 = B_2 S_2 \rightarrow B_3 = \frac{B_2 S_2}{S_3} = \frac{B_0}{4} \rightarrow H_3 = 370 \frac{A}{m}$

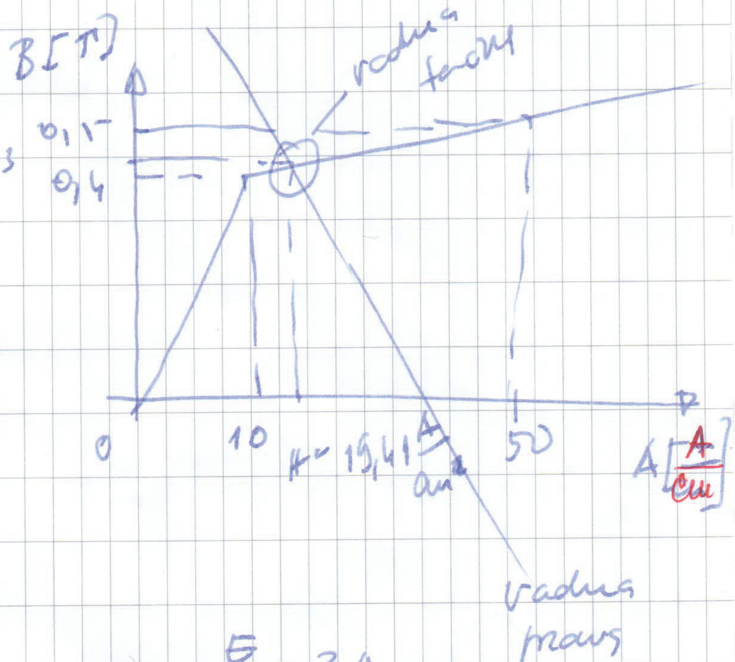
$\Phi_4 = \Phi_3 \Rightarrow B_4 S_4 = B_3 S_3 \Rightarrow B_4 = \frac{B_3 S_3}{S_4} = \frac{1}{3} B_0 \rightarrow H_4 = 410 \frac{A}{m}$

$\Phi_5 = \Phi_4 \quad B_5 S_5 = B_4 S_4 \Rightarrow B_5 = \frac{B_4 S_4}{S_5} = \frac{1}{2} B_0 \rightarrow H_5 = 540 \frac{A}{m}$ (**)

(**) u (*) $\Rightarrow$ **NI = 17,2 Anav**

1. Na tankom jezgru od feromagnetnog materijala ravnomjerno i gusto je namotano $N=1000$ namotaja tanke žice, ukupne otpornosti $R=50\Omega$. Poznato je $l=1m$, $l_0=3,14\,mm$, $S=S_0=1\,cm^2$ i $E=150V$. Idealizovana karakteristika prvobitnog magnetisanja materijala jezgra je data na slici. Magnetno rasipanje se može zanemariti. Izračunati jačinu magnetnog polja u jezgru i vazdušnom procjepu





homogeno polje

A.Z. $\quad H \cdot l + H_0 l_0 = NI \qquad I = \frac{E}{R} = 3A$

$B_0 = B_A$

$$H \cdot l + \frac{B_0}{\mu_0} l_0 = NI$$

$$H + B\,2500 = 3000$$ J-na radne prave

$H=0 \quad B=1,2$

$B=0 \quad H = 3000\,\frac{A}{m} = 30\,\frac{A}{cm}$

$H = 19,41\,\frac{A}{cm}$

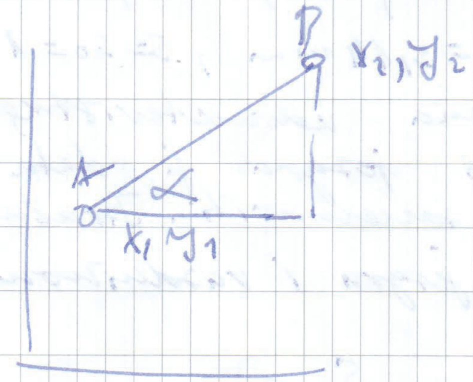
$B = 0,423\,T = B_0$

$$H_0 = \frac{B_0}{\mu_0} = 337\,\frac{kA}{m}$$

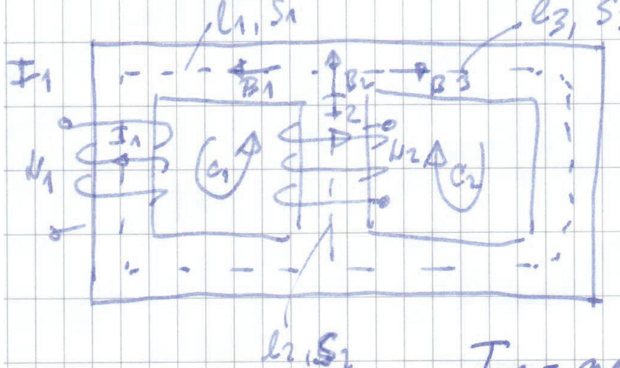
- kad imamo vazdušni procjep - linearno B - H
- isti materijali
- ista površina poprečnog presjeka

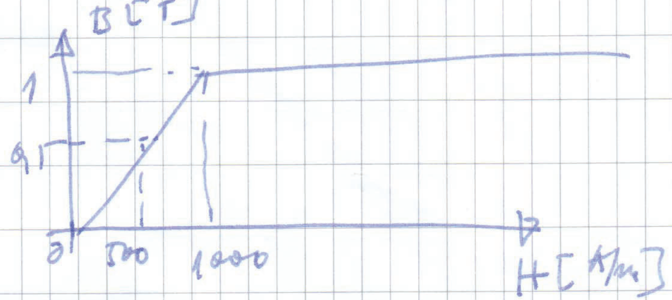
$$y - y_1 = k(x - x_1) \qquad k = tg\,\alpha$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



2. Dimenzije magnetnog kola sa slike a) su: $l_1=l_3=300mm$, $l_2=100mm$, $S_1=S_3=20cm^2$, $S_2=40cm^2$, a brojevi navojaka su: $N_1=2000$ i $N_2=5000$. Karakteristika magnetizacije materijala jezgra se može aproksimirati izlomljenom linijom ~~dat~~ prikazanom na slici b). U drugom namotaju uspostavljena je stalna struja $I_2=0,1A$. Magnetno rasipanje se može zanemariti. Kolika treba da bude jačina struje prvog namotaja da bi u grani sa tim namotajem jačina mag. polja bila jednaka 0.





I_1 - nemamo smjer. Pretpostavljamo ga

Z.o. održanja fluksa: $\Phi_2 = \Phi_1 + \Phi_3$

$B_2 S_2 = B_1 S_1 + B_3 S_3$ (1)

A. zakon. C_1: $H_2 l_2 + H_1 l_1 = N_1 I_1 + N_2 I_2$ (2)

A. zakon. C_2: $H_2 l_2 + H_3 l_3 = N_2 I_2$ (3)

3 - jne
4 nepoznate

I_1, B_1, B_2, B_3

po uslovu zadatka $H_1 = 0 \Rightarrow B_1 = 0$

iz (1) $\Rightarrow B_2 S_2 = B_1 S_1 + B_3 S_3 \Rightarrow B_2 S_2 = B_3 S_3 \Rightarrow B_2 = \frac{B_3 S_3}{S_2}$

Gledamo zasićenje: $B_2 < B_3$ → zasićenje može da nastane u B_3

$B_2 = \frac{1}{2} B_3$

Pretpostavka da je $B_3 = 1T$ zasićenje

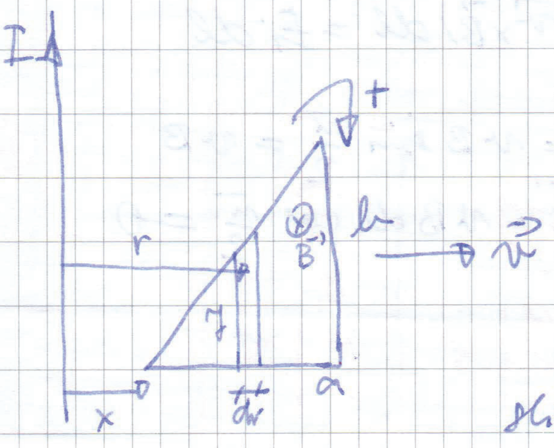
$B_2 = \frac{B_3}{2} = 0,5T$ a $H_2 = 500$ A/m sa slike

iz j-ne 3 slijedi da je: $H_3 = \frac{N_2 I_2 - H_2 l_2}{l_3} = 1500$ A/m

1500 A/m > 1000 A/m
pretpostavka je tačna

Iz. (2) $\Rightarrow I_1 = -225$ A

6.1. Provodna kontura u obliku pravouglog trougla datih dimenzija, udaljava se brzinom $\vec{v}$ od dugog pravolinijskog provodnika sa strujom I, ostajući u istoj ravni sa njim. Odrediti indukovanu ems u konturi u funkciji rastojanja x od provodnika. Zadatak riješiti na dva načina:
a) primjenom Faradejevog zakona i b) polazeći od izraza za dinamičku el. indukciju



a) $e = -\frac{d\Phi}{dt}$

$d\Phi = \vec{B}\,d\vec{s}$; $B = \frac{\mu_0 I}{2\pi r}$

$d\Phi = \frac{\mu_0 I}{2\pi r} y \cdot dr$

Sličnost trouglova: $\frac{y}{r-x} = \frac{b}{a}$

$y = \frac{b}{a}(r-x)$

$d\Phi = \frac{\mu_0 I}{2\pi r} \frac{b}{a}(r-x)\,dr$

$\Phi = \int d\Phi = \frac{\mu_0 I b}{2\pi a} \int_x^{a+x} \frac{r-x}{r}\,dr$

$\Phi = \frac{\mu_0 I b}{2\pi a} \int \frac{r}{r}\,dr - \int \frac{x}{r}\,dr = \frac{\mu_0 I b}{2\pi a}\left(r\Big|_x^{a+x} - x \ln r\Big|_x^{a+x} \right)$

$\Phi = \frac{\mu_0 I b}{2\pi a}\left(a - x \ln \frac{a+x}{x} \right)$

$e = -\frac{d\Phi}{dt}$ $\Bigg|$ $v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v}$ $\Bigg| \Rightarrow e = -\frac{d\Phi}{dx} \cdot v$

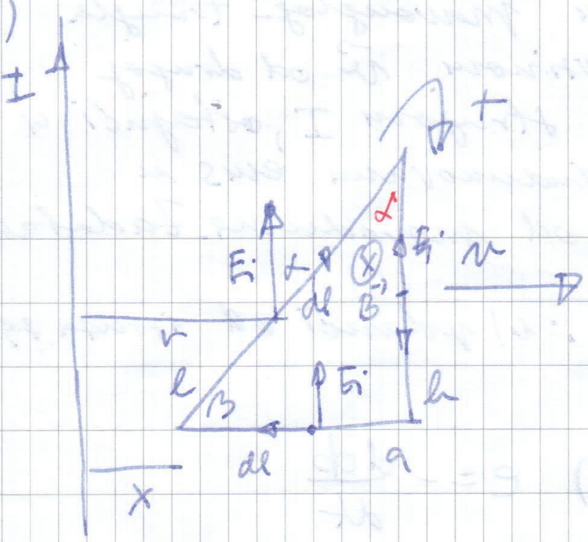
$e = \frac{\mu_0 I b \cdot v}{2\pi a} \frac{d\left(-x \ln \frac{a+x}{x}\right)}{dx} = \frac{\mu_0 I b v}{2\pi a}\left(\ln \frac{a+x}{x} - \frac{a}{a+x} \right)$

$x > 0$; $\frac{a+x}{x} > 1$ $\ln \frac{a+x}{x} > 0$

$e > 0$

$\frac{a}{a+x} < 1$

b)



$$de = \vec{dl}(\vec{v} \times \vec{B})$$

Stranica a:

$$de_a = (\vec{v} \times \vec{B})\,\vec{dl} = \vec{E}_i\,\vec{dl}$$

$$\vec{E}_i = \vec{v} \times \vec{B} = vB \sin\frac{\pi}{2} = vB$$

$$de_a = \vec{E}_i \cdot \vec{dl} = vB\,dl \cos\frac{\pi}{2} = 0$$

$$\underline{e_a = 0}$$

Stranica b:

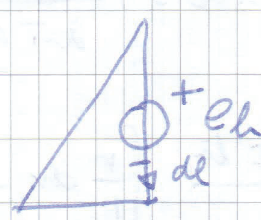
$$de_b = (\vec{v} \times \vec{B})\vec{dl} = \vec{E}_i\,\vec{dl}$$

$$E_i = \vec{v} \times \vec{B} = vB \sin \pi/2 = vB$$

$$de_b = E_i\,dl \cdot \cos \pi = -E_i\,dl = -vB\,dl$$

$$de_b = -v \frac{\mu_0 I}{2\pi(a+x)} \cdot dl$$

$$e_b = \int_b de_b = \frac{-\mu_0 I v}{2\pi(a+x)} \cdot b$$



Stranica c: $de_c = (\vec{v} \times \vec{B})\,dl$; $E_i = vB$

$$de_c = vB \cdot dl \cdot \cos\alpha = ~~vB \cdot dl~~$$

$$\cos\beta = \frac{r-x}{l} \quad ; \beta = \frac{\pi}{2} - \alpha$$

$$r - x = l \sin\alpha \quad / d$$

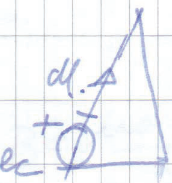
$$dr = dl \sin\alpha$$

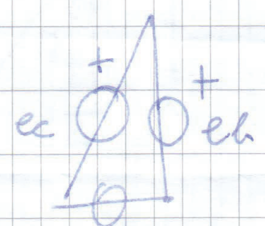
$$dl = \frac{dr}{\sin\alpha}$$

$$de_c = \frac{\mu_0 I v}{2\pi r}\cos\alpha\,\frac{dr}{\sin\alpha} = \frac{\mu_0 I v}{2\pi\,\text{tg}\,\alpha}\,\frac{dr}{r}$$

$$e_c = \int_x^{a+x} de_c = \frac{v\mu_0 I}{2\pi\,\text{tg}\,\alpha}\ln\frac{a+x}{x} > 0$$

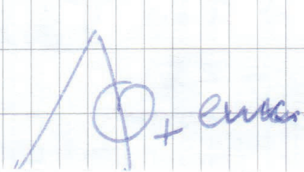
$$\text{ctg}\,\beta = \text{tg}\,\alpha = \frac{a}{b} \Rightarrow \qquad e_c = \frac{v\mu_0 I\cdot b}{2\pi a}\ln\frac{a+x}{x}$$





$$e_{mn} = e_a + e_b + e_c$$

$$e_{mn} = \frac{\mu_0 I b}{2\pi a}\left(\ln\frac{a+x}{x} - \frac{a}{a+x}\right) > 0$$



$$x\cdot\ln\frac{a+x}{x} = \ln\frac{a+x}{x} + x\cdot\frac{x}{a+x}\left(\frac{a}{x}+1\right)'$$

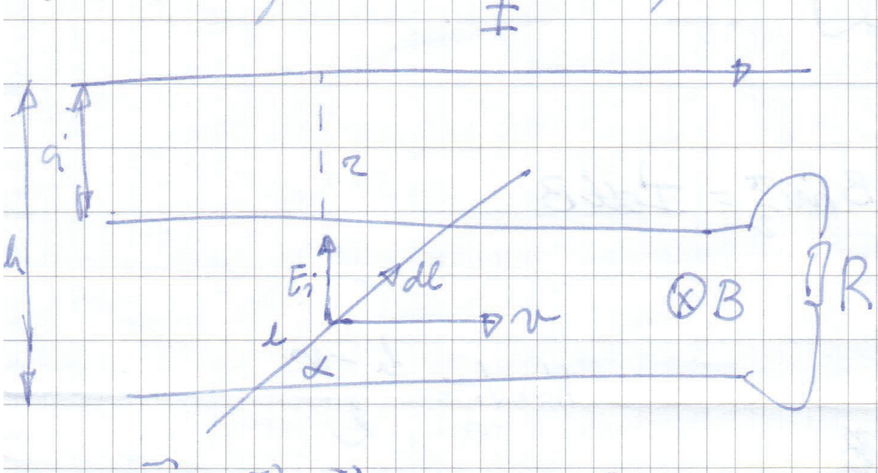
$$\ln\frac{a+x}{x} + x\,\frac{x}{a+x}\qquad a\cdot x^{-1}$$
$$-a x^{-2}$$

$$\ln\frac{a+x}{x} - \frac{x^2}{a+x}\,a x^{-2}$$

$$\boxed{\ln\frac{a+x}{x} - \frac{a}{\ldots}}$$

6.15. Veoma dugi provodnik sa strujom I nalazi se u istoj ravni sa dvije paralelne provodne šine koje se nalaze na rastojanjima a i b od njega. Preko šina se bez trenja kreće stalnom brzinom v provodni štap zaklapajući sa šinama ugao α. Šine su zatvorene na jednom kraju preko otpornosti R. Zanemarujući otpornost štapa i šina, odrediti:

a) Struju kroz otpornost R; b) mehaničku silu kojom treba djelovati da bi se provodnik kretao stalnom brzinom v; c) pokazati da je snaga koja je potrebna za održavanje ravnomjernog kretanja štapa, jednaka snazi Džulovih gubitaka u otpornosti R.



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{E_i} = \vec{v} \times \vec{B} = vB \sin\frac{\pi}{2} = vB$$

$$de = \vec{E_i}\,\vec{dl} = E_i\,dl \cos\left(\frac{\pi}{2} - \alpha\right) = E_i\,dl \sin\alpha$$

$$de = v\frac{\mu_0 I}{2\pi r}\,dl \sin\alpha \qquad \sin\alpha = \frac{b-r}{l}$$

$$b - r = l \sin\alpha \quad /d$$

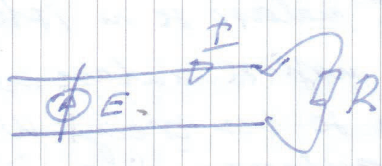
$$-dr = dl \sin\alpha$$

$$dl = -\frac{dr}{\sin\alpha}$$

$$de = -v\frac{\mu_0 I}{2\pi r}\frac{dr}{\sin\alpha}\sin\alpha$$

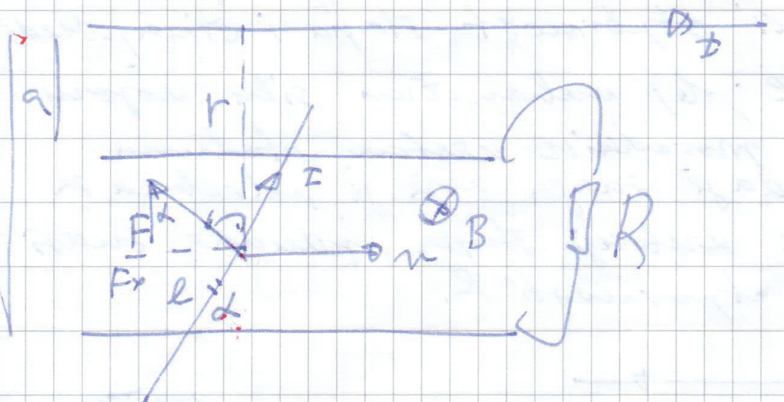
$$de = -v\frac{\mu_0 I}{2\pi}\frac{dr}{r}$$

$$e = \int dl = -\frac{\mu_0 I v}{2\pi}\int_b^a \frac{dr}{r} = -\frac{\mu_0 I v}{2\pi}\ln\frac{a}{b} = \frac{\mu_0 I v}{2\pi}\ln\frac{b}{a}$$



$$I' = \frac{e}{R} = \frac{\mu_0 I v}{2\pi R} \ln \frac{b}{a}$$

b)



$$d\vec{F} = I' d\vec{l} \times \vec{B} = I' dl\, B \sin\frac{\pi}{2} = I' dl\, B$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\sin\alpha = \frac{b - r}{l}$$

$$dF = \frac{\mu_0 I I'}{2\pi r} dl$$

$$b - r = l \sin\alpha$$

$$-dr = dl \sin\alpha$$

$$dF = \frac{\mu_0 I I'}{2\pi} \frac{-dr}{\sin\alpha \cdot r}$$

$$dF = -\frac{\mu_0 I I'}{2\pi \sin\alpha} \frac{dr}{r} \Rightarrow F = \int_b^a dF = -\frac{\mu_0 I I'}{2\pi \sin\alpha} \ln\frac{a}{b}$$

$$F_x = F \sin\alpha = \frac{\mu_0 I I'}{2\pi} \ln\frac{b}{a}$$

c)

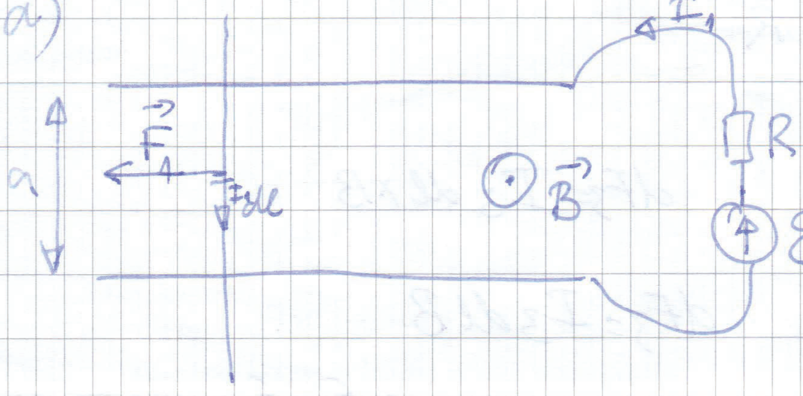
$$P_{meh} = F_{meh}\, v = \frac{\mu_0 I I'}{2\pi} \ln\frac{b}{a} \cdot v \cdot \frac{R}{R}$$

$$I' = \frac{\mu_0 I v}{2\pi R} \ln\frac{b}{a}$$

$$P_{meh} = I' I' R = R I'^2$$

6.18. Provodne šine na međusobnom rastojanju a povezane su na jednom kraju preko izvora ems $\mathcal{E}$ i otpornosti R. Na šine je, normalno na njih, oslonjen provodni štap. Zanemarujući otpornosti štapa i šina odrediti el. silu na štap po intenzitetu, pravcu i smjeru ako:
a) štap miruje; b) kreće se brzinom v u smjeru s lijeva na desno i c) kreće se istom brzinom ali u suprotnom smjeru.
Brojni podaci: $\mathcal{E}=1{,}5\,V$, $R=2\,\Omega$, $a=0{,}5\,cm$, $v=10\,\frac{m}{s}$, $B=0{,}5\,T$

a)

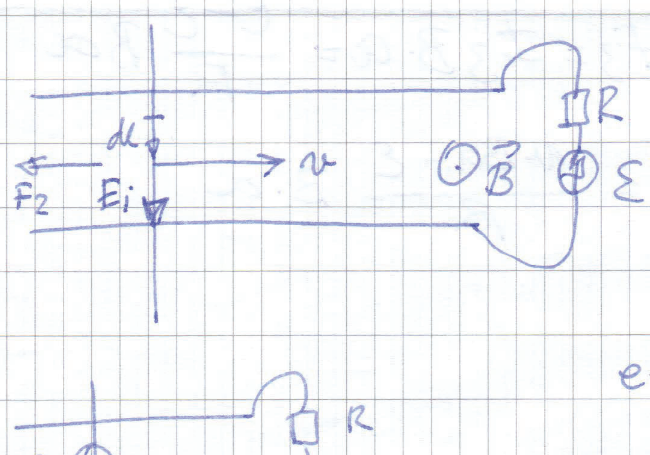


$$I_1=\frac{\mathcal{E}}{R}$$

$$d\vec{F}_1=I_1\,d\vec{l}\times\vec{B}$$

$$F_1=I_1B\cdot a=\frac{\mathcal{E}}{R}B\cdot a$$

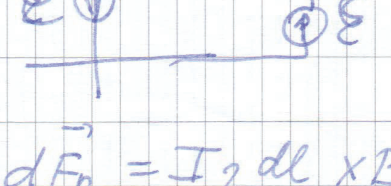
b)



$$\vec{E}_i=\vec{v}\times\vec{B}$$

$$de=\vec{E}_i\,d\vec{l}=E_i\,dl\cos 0=E_i\,dl$$

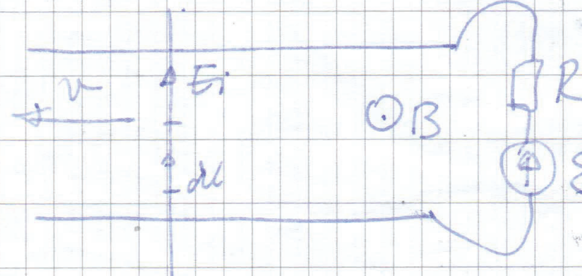
$$e=\int de=E_i\cdot a=vBa$$



$$I_2=\frac{\mathcal{E}+e}{R}=\frac{\mathcal{E}+vBa}{R}$$

$$d\vec{F}_2=I_2\,dl\times B=I_2\,dl\,B\Rightarrow F_2=\frac{\mathcal{E}+vBa}{R}B\cdot a$$

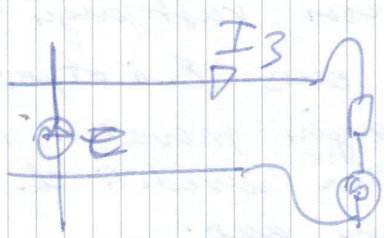
c)



$$E_i=v\times B=vB$$

$$de=E_i\,dl=E_i\,dl\cos 0=E_i\,dl$$

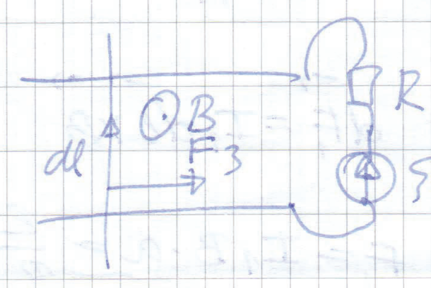
$$de=vB\,dl\Rightarrow \boxed{e=vBa}$$



$e = vBa = 10 \cdot 0{,}5 \cdot 0{,}5 = 2{,}5V$

$\mathcal{E} = 1{,}5V$

$e > \mathcal{E}$ $\qquad I_3 = \frac{e - \mathcal{E}}{R}$



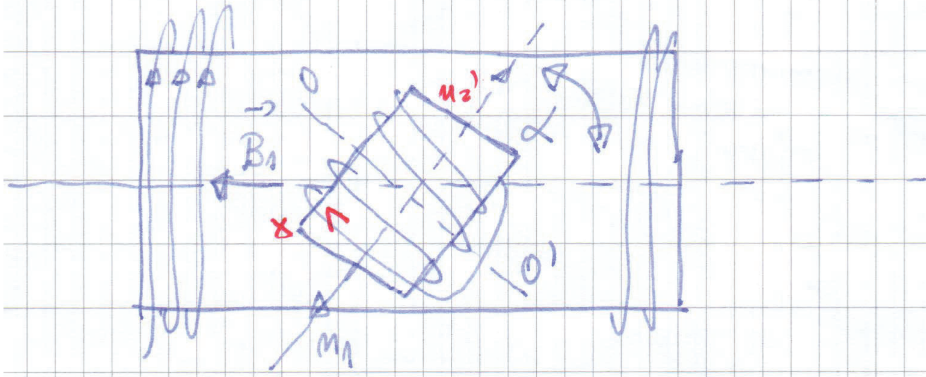
$dF_3 = I_3\, dl \times B$

$dF_3 = I_3\, dl\, B$

$F_3 = I_3 B \cdot a = \frac{e - \mathcal{E}}{R} B a$

$F_3 = \frac{vBa - \mathcal{E}}{R} B \cdot a$

6.21 - Unutar dugog solenoida dužine l_1 sa N_1 namotaja sa strujom I_1 nalazi se manji solenoid sa kratko spojenim namotajem od N_2 navojaka, i površine poprečnog presjeka S_2. Ukupna otpornost namotaja unutrašnjeg solenoida je R a ose solenoida zaklapaju ugao α. Kolika količina elektriciteta i u kojem smjeru će proteći kroz namotaj unutrašnjeg solenoida ako se on okrene oko ose OO' za $180°$



$$e = Ri = -\frac{d\Phi}{dt}$$

$$i = -\frac{1}{R}\frac{d\Phi}{dt}$$

$$\frac{dq}{dt} = -\frac{d\Phi}{dt\,R}$$

$$B_1 = \frac{\mu_0 N_1 I_1}{l_1} \qquad q = -\frac{\Delta\Phi}{R} = -\frac{\Phi'' - \Phi'}{R}$$

$$\Phi' = \vec{B}_1 \cdot S_2 \vec{n}_1 N_2 = N_2 \vec{B}_1 \cdot \vec{n} S$$

$$\Phi' = B_1 S_2 N_2 \cos\alpha$$

$$\Phi'' = \vec{B}_1 S_2 N_2 \vec{n}_2 = B_1 S_2 N_2 \cos(\pi - \alpha) = -B_1 S_2 N_2 \cos\alpha$$

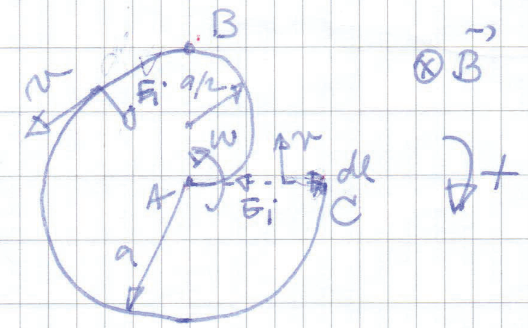
$$\Phi'' = -\Phi'$$

$$q = -\frac{\Phi'' - \Phi'}{R} = \frac{2\Phi'}{R} = \frac{2 B_1 S_2 N_2 \cos\alpha}{R}$$

$$q = \frac{2 S_2 N_2 \cos\alpha\, \mu_0 N_1 I_1}{R\, l_1}$$

Ako je $q > 0$ onda je pozitivan smjer

62. Provodnik ABC kao na slici obrće se ugaonom brzinom w oko tačke A u homogenom magnetnom polju indukcije B čije su linije normalne na ravan u kojoj leži provodnik. Odrediti indukovanu ems u provodniku. Kolike su indukovane ems u dijelovima AB i BC provodnika? Ako se uzme da je potencijal tačke A jednak 0, kolika su potencijali tačaka B i C?



kad spojimo konturu

$$\frac{d\phi}{dt} = 0 \qquad \phi = const.$$

$$E_{uk} = 0 \qquad E_{AB} + E_{BC} + E_{CA} = 0$$

Kontura BC:

$$E_i = \vec{v} \times \vec{B} = vB$$

$$de_{BC} = \vec{E_i} \cdot \vec{dl} = E_i \, dl \cos\frac{\pi}{2} = 0$$

$$\boxed{e_{BC} = 0} \Rightarrow e_{AB} = -E_{CA}$$

$$E_i = v \times B = vB = \omega \cdot r B$$

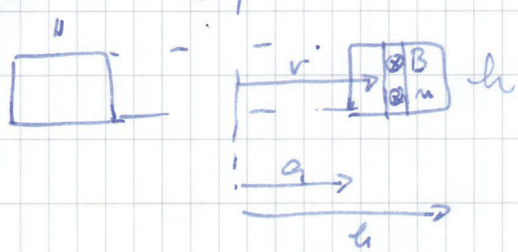
$$de_{\substack{AC \\ CA}} = \vec{E_i} \cdot \vec{dl} = \omega r B \, dl \cos\pi = -\omega r B \, dl$$

$$dl = dr$$

$$e_{\substack{AC \\ CA}} = \int_0^a -\omega r B \, dr = -\omega B \frac{a^2}{2}$$

suprotno dl!

7.2. Odrediti induktivnost namotaja od N navojaka gusto namotanih na torusnom jezgru pravougaonog presjeka. Dimenzije jezgra su date na slici, a njegova relativna permeabilnost je $\mu r = const$. Zadatak riješiti primjenom definicionog obrasca $L = \frac{\Phi}{I}$ i preko magnetne energije.



$$\oint H dl = NI$$

$$H\, 2r\pi = NI$$

$$\boxed{H = \frac{NI}{2r\pi}}$$

$\Phi = LI$, L - sopstvena induktivnost.

$$B = \mu \cdot H = \frac{\mu NI}{2r\pi}$$

Φ, L, C samo zavise od sredine

$$d\Phi = B \cdot ds = \frac{\mu_0 NI}{2r\pi} h \cdot dr$$

$$\Phi = \int d\Phi = \int_a^b \frac{\mu_0 NI}{2r\pi} h \cdot dr = \frac{\mu_0 NIh}{2\pi} \ln \frac{b}{a}$$ - fluks u poprečnom presjeku

Ukupan fluks. $\Phi_{un} = N\Phi = \frac{\mu N^2 I h}{2\pi} \ln \frac{b}{a}$

$$\boxed{L = \frac{\Phi_{un}}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}}$$

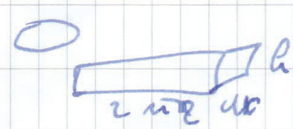
$$W_m = \frac{1}{2} LI^2 = \frac{1}{2} \Phi I = \int_V w_m dV$$ zapreminska gustina energije

$$w_m = \frac{1}{2} \vec{B}\vec{H} = \frac{1}{2} \mu H^2 = \frac{1}{2} \frac{B^2}{\mu}$$

$\boxed{B = \mu H}$ ne važi za ~~linearne~~ nelinearne

$$H = \frac{NI}{2r\pi} \qquad w_m = \frac{1}{2}\mu H^2 = \frac{1}{2}\mu \left(\frac{NI}{2r\pi}\right)^2$$

jezgro izdijelimo na tanke cilindrične ljuske

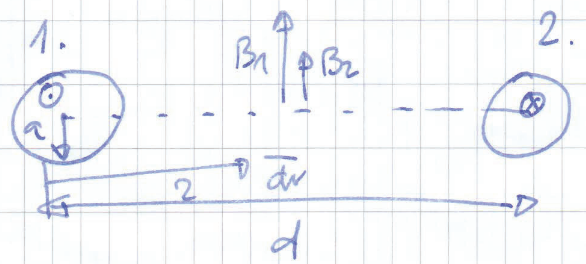


$$W_m = \int_V w_m dV = \int \frac{1}{2} \mu H^2 = \int \frac{1}{2} \mu \left(\frac{NI}{2r\pi}\right)^2 \underbrace{2r\pi \cdot h \cdot dr}_{dV}$$

$$W_m = \frac{1}{2} \mu \frac{N^2 I^2}{2\pi} h \int_a^b \frac{dr}{r} \Rightarrow$$

$$W_m = \frac{1}{2} \mu \frac{N^2 I^2}{2\pi} h \ln \frac{b}{a} = \frac{1}{2} LI^2 \Rightarrow \boxed{L = \frac{\mu N^2 h}{2\pi} \ln \frac{b}{a}}$$

7.6. Odrediti podužnu induktivnost dvožičnog voda čije su (*) dimenzije date na slici. Permeabilnost provodnika je $\mu = \mu_0$, $a = 1{,}5\,mm$, $D = 20\,cm$.



$$\oint_L H\,dl = I\,d \qquad H \cdot 2r\pi = \frac{I}{a^2\pi} \cdot r^2\pi$$

$$\boxed{H = \frac{I}{2a^2\pi} r}$$

$$W_m = \int_V w_m\,dV$$

$$W_m = \int \frac{1}{2} \mu_0 \frac{I^2 r^2}{4a^4\pi^2} \cdot 2r\pi \cdot dr \cdot l$$

$$= \frac{\mu_0 I^2 l}{4a^4\pi} \int_0^a r^3\,dr = \frac{\mu_0 I^2 l}{4a^4\pi} \frac{a^4}{4} = \frac{1}{2} L_u I^2$$

$$\boxed{L_u' = \frac{L_u}{l} = \frac{\mu_0}{8\pi}}$$

Induktivnost između provodnika

$$B_2 = B_1 + B_2 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi(d-r)}$$

$$d\phi = B \cdot dS = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r} + \frac{1}{d-r}\right) dr \cdot l$$

$$\phi = \frac{\mu_0 I}{2\pi} l \left(\int_a^{d-a} \frac{dr}{r} + \int_a^{d-a} \frac{dr}{d-r} \right)$$

$$d - r = t \qquad -dr = dt \qquad \frac{dr}{d-r} = \frac{-dt}{t}$$

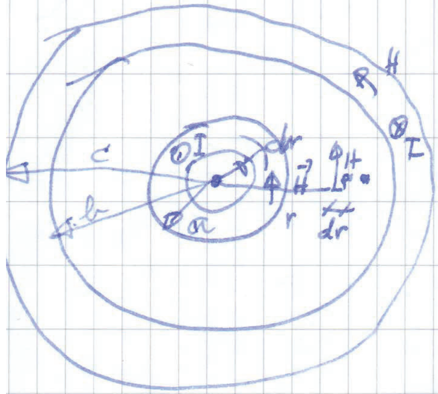
$$L_s' = \frac{\phi}{I \cdot l} = \frac{\mu_0}{2\pi}\left(\ln\frac{d-a}{a} - \ln\frac{d-(d-a)}{d-a}\right) = \frac{\mu_0}{2\pi}\left(\ln\frac{d-a}{a} - \ln\frac{a}{d-a}\right) \Rightarrow$$

$$L_s' = \frac{\mu_0}{2\pi} \ln \frac{\frac{d-a}{a}}{\frac{a}{d-a}} = \frac{\mu_0}{\pi} \ln\frac{d-a}{a} \approx a \ll d \approx \frac{\mu_0}{\pi} \ln\frac{d}{a}$$

$$L_{uk}' = 2\frac{\mu_0}{8\pi} + \frac{\mu_0}{\pi}\ln\frac{d}{a}$$

$$\boxed{L_{uk}' = 2L_u' + L_s'}$$

7.8. Odrediti podužnu induktivnost koaksijalnog kabla poznatih dimenzija. Permeabilnost provodnika je $\mu = const$. Brojni podaci: $a = 1mm$, $b = 2,5mm$, $c = 3mm$, μ_0



polje izvan kabla je 0 $\oint_L H dl = \int B ds$

1° $r < a$ $H \cdot 2r\pi = \frac{I}{a^2\pi} r^2 \pi \Rightarrow \boxed{H = \frac{I}{2a^2\pi} r}$

$$W_m = \int_V w_m dv = \int_0^a \frac{1}{2}\mu_0 \left(\frac{I}{2a^2\pi}\right)^2 r^2 \underbrace{2\pi r\, dr \cdot l}_{dv} = \frac{\mu_0 I^2 l}{4a^4\pi}\int_0^a r^3 dr \Rightarrow$$

2° $a < r < b$

Singularitet za $r=0$ ne može za nelinearne

$$\boxed{L_1' = \frac{\mu_0}{8\pi}}$$

$B = \frac{\mu_0 I}{2\pi r}$

$$d\phi = B ds = \frac{\mu_0 I}{2\pi r} l\, dr \cos 0° \Rightarrow \phi = \frac{\mu_0 I l}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln\frac{b}{a}$$

$$\boxed{L_2' = \frac{L_2}{l} = \frac{\phi}{I \cdot l} = \frac{\mu_0}{2\pi}\ln\frac{b}{a}}$$

3° $b < r < c$ $H\, 2r\pi = I - \frac{I}{(c^2-b^2)\pi}(r^2-b^2)\pi \Rightarrow H = \frac{I(c^2-r^2)}{2\pi r(c^2-b^2)}$

$\boxed{B = \mu H}$

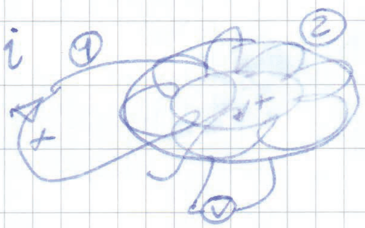
$$d\phi = \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi r}\frac{(c^2-r^2)}{(c^2-b^2)} l \cdot dr$$

$$\phi = \int d\phi = \frac{\mu_0 I l}{2\pi(c^2-b^2)}\int_b^c \frac{c^2-r^2}{r} dr = \frac{\mu_0 I l}{2\pi(c^2-b^2)}\left(c^2\ln\frac{c}{b} - \frac{1}{2}(c^2-b^2)\right)$$

$$\boxed{L_3' = \frac{\phi}{I \cdot l} = \frac{\mu_0}{2\pi(c^2-b^2)}\left(c^2 \ln\frac{c}{b} - \frac{1}{2}(c^2-b^2)\right)}$$

$$\boxed{L_{uk}' = L_1' + L_2' + L_3'}$$

7.12. Tanak torus, srednje dužine l_{sr}, površine pop. presjeka S i sa N namotaja, obuhvaćen je zatvorenim provodnikom proizvoljnog oblika. Kroz provodnik protiče prostoperiodična struja nepoznate amplitude $i = I_m \cos 314t$. Na krajeve namotaja je priključen voltmetar koji mjeri amplitudu prostoperiodične indukovane ems u namotaju. Kolika je amplituda I_m struje u provodniku, ako je amplituda indukovane ems $E_m = 1,6\,mV$, $l_{sr} = 60\,cm$, $S = 1,5\,cm^2$, $N = 800$, $\mu = \mu_0$



$i = I_m \cos \omega t \qquad \omega = 2\pi f$

$\omega t = 314 t$

$$e = -\frac{d\phi}{dt} = -\frac{d\phi_{12}}{dt} = -L_{12}\frac{di}{dt} \qquad \boxed{\phi = L i}$$

$\phi_{12} \Rightarrow$ fluks koji stvara kontura kroz torus $\quad \phi_{12} = L_{12} i_1$

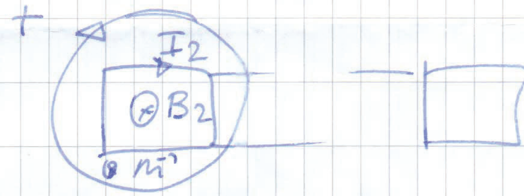
$\boxed{L_{12} = L_{21}}$

tanak torus

$$B_2 = \frac{\mu_0 N I_2}{l_{sr}}$$

$H l_{sr} = N I$

$B = \mu_0 H$



$$\phi_{21} = \frac{\mu_0 N I_2}{l_{sr}} S$$

$$L_{21} = \frac{\phi_{21}}{I_2} = \frac{\mu_0 N S}{l_{sr}} = L_{12}$$

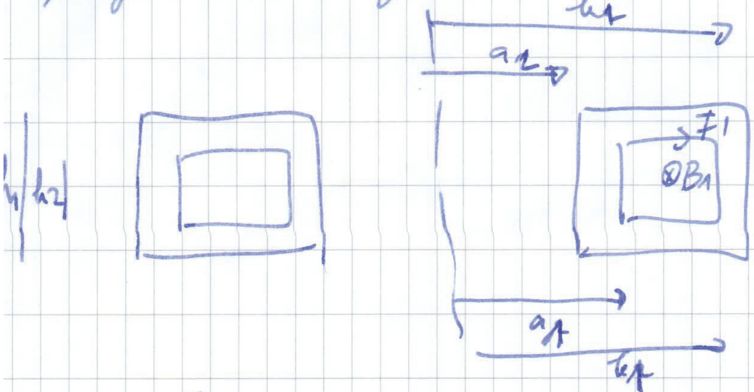
$$e = -\frac{d\phi_{12}}{dt} = -L_{12}\frac{di}{dt} = -L_{21}\frac{di_1}{dt}$$

$$= -\frac{\mu_0 N S}{l_{sr}} \frac{d(I_m \cos 314t)}{dt} = \frac{\mu_0 N S}{l_{sr}} I_m 314 \sin 314t$$

$E = E_m \sin 314 t \qquad E_m = \dfrac{\mu_0 N S I_m \cdot 314}{l_{sr}}$

$$I_m = \frac{E_m \cdot l_{sr}}{\mu_0 N S \cdot 314} = 20{,}27\,A$$

7.15. Dva kartonska torusa pravougaonog poprečnog presjeka postavljena su koaksijalno jedan u drugom. Broj namotaja na unutrašnjem torusu je N_1, a na spoljašnjem N_2, dok su dimenzije torusa prikazane na slici. Odrediti koeficijent sprege namotaja u torusu.



mjera spregnutosti sistema

$$k = \frac{|L_{12}|}{\sqrt{L_1 L_2}}$$

posmatramo prvi torus kao da nema drugog

$$L_1 = \frac{\Phi_1}{I_1} \qquad d\Phi_1' = \vec{B}\,d\vec{S} \Rightarrow \quad \Phi_1' = \frac{\mu_0 N_1 I_1 h_1}{2\pi}\ln\frac{b_1}{a_1}$$

$$\Phi_1 = N_1 \Phi_1' = \frac{\mu_0 N_1^2 I_1 h_1}{2\pi}\ln\frac{b_1}{a_1}$$

$$L_1 = \frac{\Phi_1}{I_1} = \frac{\mu_0 N_1^2 h_1}{2\pi}\ln\frac{b_1}{a_1} \Longleftrightarrow L_2 = \frac{\mu_0 N_2^2 h_2}{2\pi}\ln\frac{b_2}{a_2}$$

analogno

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propustimo struju kroz prvi namotaj: U drugom namotaju ćemo dobiti fluks

polje postoji samo unutar prvog torusa

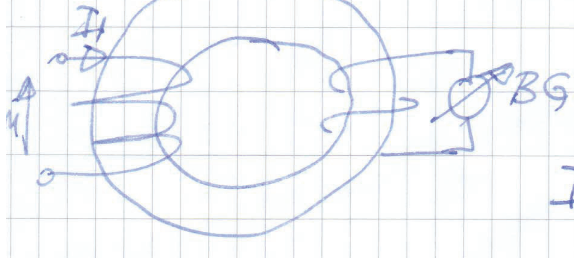
$$\Phi_{12}' = B_1 dS = B_1 dr \cdot h_1$$

$$\Phi_{12}' = \frac{\mu_0 N_1 I_1 h_1}{2\pi}\ln\frac{b_1}{a_1} \qquad \Phi_{12} = N_2 \Phi_{12}' = \frac{\mu_0 N_1 N_2 I_1 h_1}{2\pi}\ln\frac{b_1}{a_1}$$

$$L_{12} = \frac{\Phi_{12}}{I_1} = \frac{\mu_0 N_1 N_2}{2\pi}\ln\frac{b_1}{a_1}$$

$$k = \frac{|L_{12}|}{\sqrt{L_1 L_2}} = \frac{h_1 \ln\frac{b_1}{a_1}}{\sqrt{h_1 h_2 \ln\frac{b_1}{a_1} \ln\frac{b_2}{a_2}}}$$

7.19. Data su dva spregnuta namotaja induktivnosti L_1 i L_2. Prvi namotaj je priključen na stalni napon U, dok su krajevi drugog vezani za balistički galvanometar. Otpornost prvog namotaja je R_1, a ukupna otpornost drugog namotaja i BG je R_2. Kada se ukine struja kroz prvi namotaj, tada BG pokaže da je kroz drugi namotaj protekla ~~struja~~ Q. Koliki je koeficijent sprege kalemova.
Brojni podaci: $U=10\,V$, $R_1=1\,\Omega$, $L_1=1\,mH$, $L_2=4\,mH$, $R_2=2\,\Omega$, $Q=8{,}5\,mC$.



$$k=\frac{|L_{12}|}{\sqrt{L_1L_2}}$$

$$I_1=\frac{U}{R_1}$$

$$Q=-\frac{\Delta\Phi}{R_2}=-\frac{\Phi''-\Phi'}{R_2}=\frac{\Phi'}{R_2}$$ $\Phi''=0$ jer je ukinuta struja

$$\Phi'=QR_2=\Phi_{12} \qquad \Phi_{12}=L_{12}I_1$$

$$L_{12}I_1=QR_2$$

$$L_{12}=\frac{QR_2R_1}{U}$$

$$k=\frac{|L_{12}|}{\sqrt{L_1L_2}}=\frac{QR_1R_2}{U\sqrt{L_1L_2}}=0{,}85$$